A Simple Open Economy Model: A Non-Linear Dynamic Approach

Jan Kodera – Tran Van Quang[*]

Abstract:

The objective of this article is to derive a simple dynamic macroeconomic model of an open economy to show how an economy as a dynamic system can work. The proposed model is resulted from the traditional Mundell-Fleming model. Unlike the Mundell-Fleming model, we introduce a continuous dynamic and non-linearity. Non-linearity in our model is represented by a non-linear investment function. The non-linear investment function is introduced as the propensity to invest function, which is assumed to be captured by the logistic function of production. After that, the stability of the model is analysed using Hurwitz stability theorem. The behaviour of our non-linear macroeconomic model of open economy is demonstrated on two numerical examples in which two different sets of parameters are selected to examine the dynamic of the system with emphasis on the impact of export multiplier. The presented examples show that the model is able to generate very complex dynamic.

Key words: Dynamic model; Money market dynamics; Uncovered interest rate parity; Exchange rate dynamics; Limit cycle.

JEL classification: E44.

1 Introduction

The Mundell Fleming model has been a very important model of international economics (Mundell, (1963, 1962), Fleming, (1962)). Though there are other alternatives that have appeared later, it still is a useful tool for policy analysis of an open economy (Obstfeld and Rogov, 1996). The model is a system where the balance of payment curve (BP curve) is added to a standard system of IS and LM curve. It can be used to demonstrate an intuitive dynamics of the production, interest rate and exchange rate. It provided essential background for the well-known Dornbusch model (1980), in which a more precise demonstration of price and exchange rate dynamics is studied. In Dornbusch's model the equilibrium is the saddle point equilibrium. The principles of the rational expectation are the reason why the system moves on the stable manifold. A further development of

[*] Jan Kodera; University of Economics in Prague, Faculty of Finance and Accounting, Department of Monetary Economics, W. Churchill Sq. 4, 130 67 Prague 3, Czech Republic, <kodera@vse.cz>.

Tran Van Quang; University of Economics in Prague, Faculty of Finance and Accounting, Department of Monetary Economics W. Churchill Sq. 4, 130 67 Prague 3, Czech Republic, <tran@vse.cz>.

The article is processed as an output of a research project *Institutional Support* of Faculty of Finance and Accounting, University of Economic, Prague under the registration number IP 100040.

the Mundell Fleming model can be found for example in works of Turnovsky (2000) or Flaschel et al (1997) who propose models of a simple open economy based on the assumption that a representative consumer chooses his equilibrium by solving an inter-temporal optimization problem. In our contribution, we derive a continuous time dynamic version of the original Mundell-Fleming model. Our model as such is a dynamic system which creates a framework for us to study the stability of the system. By doing so, we try to show that nonlinearity in a macroeconomic endogenous model can considerably change the dynamic of the system from a relatively simple nature towards to a more complex one. For this purpose, we have formulated the investment function in form of decreasing function of interest rate and a logistic function of production. Finally, we shore up our theoretical debate with two numerical examples to show what dynamics can be generated by our model. The article is structured exactly as intention stated above. We also include the Matlab code used to solve our model in the appendix at the end of the paper for a possible reproduction.

2 The Model Formulation

This simple open economy model presented in this article is originated from Mundell-Fleming static IS – LM model for an open economy with fixed price level. Our interest is to expand the scope with respect to commodities and the money market. In this paper we revitalize the approach used to formulate the model in previous Kodera's works (Kodera et al., 2007, Sladký et al., 1999). A dynamics of the production Y is described by the following equation

$$\dot{Y} = \alpha[I(Y,R) + X(Z) - S(Y,R)], \tag{1}$$

It is noticeable that the symbol t for time is omitted but all variables in our model are function of time. The expression in brackets of equation (1) is the difference between the demand and supply. The demand of an economy without public sector is $C + I + E$ where C, I, E denote consumption, the demand for investment, the demand for export respectively and the supply is defined as a total of $C + S + M$ where C, S, M, is consumption, savings, import respectively. Subtracting the supply from the demand we get $I + X - S$, where $X = E - M$ denotes net export. The difference between the demand and the supply causes the increase of production if the difference is positive and it decreases when it is negative. The dynamics of production could be re-formulated dividing equation (1) by Y what yields

$$\frac{\dot{Y}}{Y} = \alpha\left[\frac{I(Y,R)}{Y} + \frac{X(Z)}{Y} - \frac{S(Y,R)}{Y}\right]. \tag{2}$$

The investment function in our model is supposed to have the form

$$I = I(R,Y). \tag{3}$$

Without losing generality this investment function is assumed to be non-linear, increasing in Y and decreasing in R. Expression $I(Y,R)/Y$ is often called the propensity to invest. Denoting $\log Y = y$ we rewrite propensity to invest as $I(e^y, R)e^{-y}$ or more simply

$$\frac{I(Y,R)}{Y} = i(y,R), \tag{4}$$

where $i(.,R) = I(\exp(.),\mathrm{R})/\exp(.)$. Function i is assumed to be increasing in y. Expressions $X(Z)/Y$ and $S(Y,R)/Y$ denote propensity to export and propensity to save respectively. The net export is increasing function of $z = \log Z$. If the export multiplier, which is an analogue of investment multiplier[1], is high, then the product Y will increase faster, which can result in the decline of the propensity to export when exchange rate increases. If the export multiplier is low, then the product does not growth so fast and the propensity to export decreases. Without losing generality we assume a semi-log linear relationship between propensity to export and log of exchange rate we get

$$\frac{X}{Y} = x_0 + x_3 \log Z, \tag{5}$$

where x_3 a real number, whose sign is dependent on the size of export multiplier.

Similarly, propensity to save is increasing function of variables Y, R. In order to

$$\frac{S}{Y} = s_0 + s_1 \log Y + s_2 R, \tag{6}$$

where s_0 >0, s_1>0, s_2>0. A graph of the above function restricted to Y in the plane of (Y, S) for s_0=0.12, s_1=0.8, s_2=1.6 and R=0.05 is presented in Figure 1.

Let the logarithms of variables X and Z are denoted by lower case symbols i.e.

$$x = \log X, z = \log Z. \tag{7}$$

Substituting (4)-(7) into equation (2) we get

$$\dot{y} = \alpha[i(y,R) + x_0 + x_3 z - (s_0 + s_1 y + s_2 R)]. \tag{8}$$

In order to obtain a similar setting as in the commodity market, the money market is assumed to exhibit geometrical adjustment, which in general does not differ from the arithmetical adjustment, i.e.

[1] The investment multiplier is defined as a ratio of production increase to an investment increase. The export multiplier is defined in the same way which is the ratio of production increase to an export increase.

$$e^{\dot{R}} = \left[\frac{L(Y,R,Z)}{M}\right]^{\beta}, \beta > 0. \tag{9}$$

where M and L denotes money supply and demand for money respectively. The demand for money is a function of production, nominal interest rate and exchange rate in the form of the power function often used in economic literature

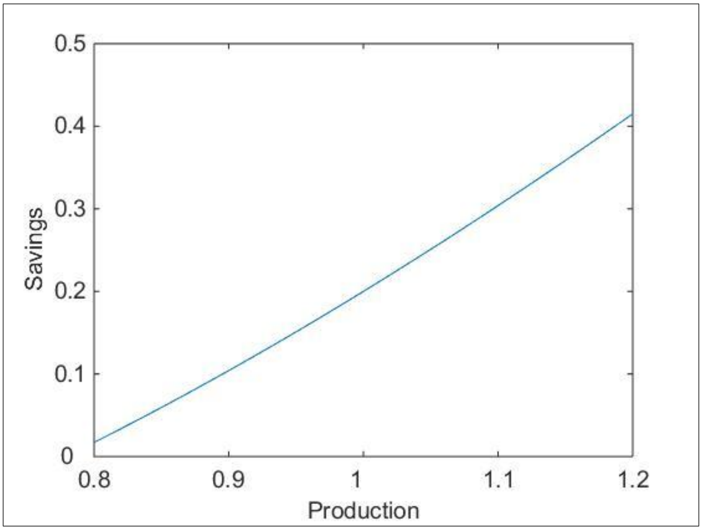
$$L(Y,R,Z) = L_0 Y^{l_1}(1+R)^{-l_2} Z^{l_3}, \tag{10}$$

where L_0, l_1, l_2 are positive numbers. After taking a logarithm of equation (9) we get

$$\dot{R} = \beta[l_0 + l_1 y - l_2 R + l_3 z - m], \tag{11}$$

where l_0 =log L_0, m=log$M$.

Fig. 1 The saving function



Source: Data source (authorial computation).

As for equation (11) it is obvious that l_3 is positive because of the assumption that the domestic money and foreign money are considered to be strong substitutes. The third equation of the model is the one that describes exchange rate dynamics and is based on uncovered interest rate parity condition (Mussa, 1986). The uncovered interest rate parity is the following relation

$$Z(t+h) = Z(t)\frac{1+hR}{1+hR^*}, \tag{12}$$

where h is the increase of time, R is domestic interest rate and R^* is foreign interest rate at time t. Taking logarithm, dividing by h and letting $h \to 0$ we get

$$\dot{z} = R - R^*. \tag{13}$$

The model consists of equations (8), (11) and (13). This system of equations has the following form

$$\begin{gathered} \dot{y} = \alpha[i(y,R) + x_0 - x_3 z - (s_0 + s_1 y + s_3 R)], \\ \dot{R} = \beta[l_0 + l_1 y - l_2 R + l_3 z - m], \\ \dot{z} = R - R^*. \end{gathered} \tag{14}$$

3 Introducing Non-Linearity into the System

In this section we introduce non-linearity into the system of equations. It can be done through the only non-linear function in system (14) called the propensity to invest which is a non-linear function of logarithm of product and interest rate. Let us assume that this propensity to invest is multiplicatively separable hence it is a product of two one-variable functions as follows

$$i(y,R) = h(y)g(R). \tag{15}$$

Let function g be a reciprocal function of interest rate factor $1 + R$:

$$g = \frac{1}{1+R}, \tag{16}$$

and as propensity to invest increases with y and should have an infimum and supremum, hence a logistic function of logarithm product should be a very suitable candidate for h. Let us choose the following form of the logistic function:

$$h(y) = \frac{i_0}{1+\exp(i_1 - i_2 y)}, \tag{17}$$

where $i_\mu, \mu = 0,1,2$ are parameters of logistic function and $i_\mu > 0, \mu = 0,2$.

The analysis of the non-linear system will be done through the following system

$$\begin{gathered} \dot{y} = \alpha\left[\frac{1}{1+R}\frac{i_0}{1+e^{i_1 - i_2 y}} + x_0 + x_3 z - (s_0 + s_1 y + s_2 R)\right], \dot{R} = \beta[l_0 + l_1 y - l_2 R + l_3 z - m^s], \\ \dot{z} = R - R^*. \end{gathered} \tag{18}$$

Before linearizing this three-dimensional system of open economy above we introduce stationary state of system (18) which is the state where the system is solved for constant values of product, interest rate and exchange rate.

$$\begin{gathered} 0 = \alpha\left[\frac{1}{1+\bar{R}}\frac{i_0}{1+e^{i_1 - i_2 \bar{y}}} + x_3 \bar{z} - (s_0 + s_1 \bar{y} + s_2 \bar{R})\right], \\ = \beta[l_0 + l_1 \bar{y} - l_2 \bar{R} + l_3 \bar{z} - m^s], \\ 0 = \bar{R} - R^*, \end{gathered} \tag{19}$$

where $\bar{y},\ \bar{R},\ \bar{z}$ is a stationary solution the system.

Taking partial derivatives of the first, second and third equation of system (14) with respect to y, R and z in the point $\bar{y}, \bar{R},\ \bar{z}$ we get a Jacobi matrix of the system (14) is

$$J = \begin{bmatrix} \frac{\alpha}{(1+\bar{R})} \frac{i_0 i_2}{e^{-(j_1 - j_2 \bar{y})} + 2 + e^{j_1 - j_2 \bar{y}}} - \alpha s_1 & -\frac{\alpha}{(1+\bar{R})^2} \frac{i_0}{1+e^{i_1 - i_2 y}} - \alpha s_2 & \alpha x_3 \\ \beta l_1 & -\beta l_2 & \beta l_3 \\ 0 & 1 & 0 \end{bmatrix}, \tag{20}$$

Denoting

$$j_1 = \frac{1}{(1+\bar{R})} \frac{i_0 i_2}{e^{-(j_1 - j_2 \bar{y})} + 2 + e^{j_1 - j_2 \bar{y}}}, \tag{21}$$

$$j_2 = \frac{1}{(1+\bar{R})^2} \frac{i_0}{1+e^{i_1 - i_2 y}},. \tag{22}$$

The Jacobi matrix (20) will take the form

$$J = \begin{bmatrix} \alpha(j_1 - s_1) & -\alpha(j_2 + s_2) & \alpha x_3 \\ \beta l_1 & -\beta l_2 & \beta l_3 \\ 0 & 1 & 0 \end{bmatrix}. \tag{23}$$

Linearized system (18) has a form

$$\begin{bmatrix} \dot{\delta y} \\ \dot{\delta R} \\ \dot{\delta z} \end{bmatrix} = J \begin{bmatrix} \delta y \\ \delta R \\ \delta z \end{bmatrix}, \tag{24}$$

where $\delta y = y - \bar{y},\ \delta R = R - \bar{R},\ \delta z = z - \bar{z}.$

The Jacobi matrix plays important role in analysis of non-linear systems (Guckenheimer et al., 1986). If the real parts of its eigenvalues are negative, then stationary solution is asymptotically stable. If not, the stationary solution is asymptotically unstable and in the case of non-linear systems the existence of more complex dynamics is possible. There is several theorems on stability or non-stability of matrices (Hahn, 1982). One of them is Hurwitz criterion which is a necessary and sufficient condition for all roots of the characteristic polynomial to have negative real part. Hurwitz criterion states that the stationary solution to the linearized system is asymptotically stable if and only if principal minors of Hurwitz matrix are positive. Hurwitz matrix is constructed from the characteristic polynomial of Jacobi matrix (23) of the linearized system, which is

$$\det(\lambda E - A) = \det \begin{bmatrix} \lambda + \alpha(s_1 - j_1) & \alpha(i_2 + s_2) & -\alpha x_3 \\ -\beta l_1 & \lambda + \beta l_2 & -\beta l_3 \\ 0 & -1 & \lambda \end{bmatrix}, \tag{25}$$

and equalizing it to zero, we get characteristic equation

$$\begin{gathered} \lambda^3 + \lambda^2[\alpha(s_1 - j_1) + \beta l_2] + \lambda[\alpha\beta(s_1 - j)l_2 + \alpha\beta(j_2 + s_2)l_1 - \beta l_3] \\ -\alpha\beta(s_1 - j)l_3 - \alpha\beta l_1 x_3 = 0. \end{gathered} \tag{26}$$

The coefficients of the characteristic equation are used for the construction of Hurwitz matrix

$$H = \begin{bmatrix} \alpha(s_1 - j_1) + \beta l_2 & -\alpha\beta(s_1 - j_1)l_3 - \alpha\beta l_1 x_3 & 0 \\ 1 & \alpha\beta(s_1 - j_1)l_2 + \alpha\beta(j_2 + s_2)l_1 + \beta l_3 & 0 \\ 0 & \alpha(s_1 - j_1) + \beta l_2 & -\alpha\beta(s_1 - j_1)l_3 - \alpha\beta l_1 x_3 \end{bmatrix}. \quad (27)$$

According to Hurwitz Stability Criterion system (18) is stable only if principal minors $det\, H_i$ of matrix H are positive (Takayama, 1994):

$$\det H_1 = [\alpha(s_1 - j_1) + \beta l_2] > 0, \quad (28)$$

$$\det H_2 = \det \begin{bmatrix} \alpha(s_1 - j_1) + \beta l_2 & \alpha\beta(s_1 - j_1)l_3 - \alpha\beta l_1 x_3 \\ 1 & \alpha\beta(s_1 - j_1)l_2 + \alpha\beta(j_2 + s_2)l_1 + \beta l_3 \end{bmatrix} > 0, \quad (29)$$

$$\det H_3 = \det H =$$

$$\det \begin{bmatrix} \alpha(s_1 - i_1) + \beta l_2 & -\alpha\beta(s_1 - i_1)l_3 - \alpha\beta l_1 x_3 & 0 \\ 1 & \alpha\beta(s_1 - i_1)l_2 + \alpha\beta(i_2 + s_2)l_1 + \beta l_3 & 0 \\ 0 & \alpha(s_1 - i_1) + \beta l_2 & -\alpha\beta(s_1 - i_1)l_3 - \alpha\beta l_1 x_3 \end{bmatrix} > 0. \quad (30)$$

Let us mention a quite interesting problem. Parameter i_1 could be interpreted as the sensitivity of propensity to invest $i(y, R)$ to logarithm of product $y(t)$. Parameter s_1 is obviously called as sensitivity of propensity to save $s(y, R)$ to logarithm of product $y(t)$. If investors react to changes in production less than savers, i.e. $i_1 < s_1$ and export multiplier is relatively low $x_3 > 0$, then the economy described by system (18) cannot generate stable development. It can easily be shown using Hurwitz Stability Criterion. Expanding $det\, H_3$ along the third column we get

$$\det H_3 = (-1)^{3+3} [\alpha\beta(s_1 - i_1)l_3 - \alpha\beta l_1 x_3] \det H_2. \quad (31)$$

Hurwitz Criterion of Stability cannot be fulfilled because $-\alpha\beta(s_1 - i_1)l_3 - \alpha\beta l_1 x_3 < 0$ as all parameters are positive and $s_1 - i_1 > 0$ which ultimately leads to the fact that

$$\det H_3 < 0. \quad (32)$$

If $i_1 > s_1$ then sufficient conditions for unstable solution of the system (14) is

$det\, H_1 = [\alpha(s_1 - i_1) + \beta l_2] < 0,$ which is fulfilled if α is sufficiently high provided β, i_1, s_1, l_2 are given.

For our further analysis we assume $i_1 > s_1$ and α sufficiently high. We also assume the effect of change of export on change of product is relatively small, i.e. export multiplier is relatively low. As exchange rate influences net export positively, increasing exchange rate increases export. Increasing export increases product but not so much so export-production ratio increases too which results in a $x_3 > 0$. This fact will be demonstrated by a simple numerical example of our model. In this example we choose numerical values of parameters which are consistent with economic nature. After establishing matrix of the linear system (18), we choose its eigenvalues and evaluate the system's stability. To demonstrate how the model works we introduce two numerical examples for which the solution of the system is found and its phase portrait and evolution of all variables are shown in the next section.

4 Numerical examples

4.1 Example 1

Let the parameters in system (20) are the following:

$$j_0 = 0.42\,, j_1 = 0, j_2 = 10, x_0 = 0, x_3 = 0.19, s_0 = 0.12, s_1 = \ 0.8,\ s_2 = 1.6,$$
$$l_0 = 0.03, l_1 = 0.1, l_2 = 0.6, l_3 = 0.19,\ R^* = 0.05, \alpha = 20, \beta = 1, m = 0.$$

These values are chosen in such a way so that they satisfy both requirements of economic theory and results of empirical models. Form this set of parameters we select those that can provide us interesting dynamics. Then system (18) takes the form

$$\begin{gathered}\dot{y} = 20\left[\frac{1}{1+R}\frac{0.42}{1+e^{-10y}} + 0.19z - (0.12 + 0.8y + 1{,}6R)\right],\\ \dot{R} = 0.03 + 0.1y - 0.6R + 0.19z,\\ \dot{z} = R - 0.05.\end{gathered} \tag{33}$$

For stationary solution, we get

$$\begin{gathered}0 = \frac{1}{1+\bar{R}}\frac{0.42}{1+e^{-10\bar{y}}} + 0.19\bar{z} - (0.12 + 0.8y + 1.6\bar{R}),\\ 0 = 0.03 + 0.1\bar{y} - 0.6R + 0.19\bar{z},\\ 0 = \bar{R} - 0.05.\end{gathered} \tag{34}$$

It is not difficult to find that the stationary solution is: $\bar{y} = 0, \bar{R} = 0.05, \bar{z} = 0$. After substitution numerical values in Jacobi matrix (23) we have

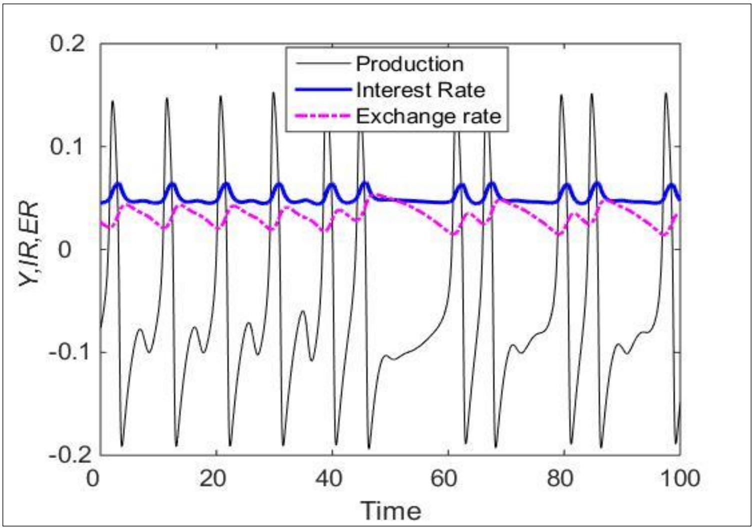
$$J = \begin{bmatrix} 4.0 & -35.8095 & 3.8 \\ 0.1 & -0.6 & 0.19 \\ 0 & 1 & 0 \end{bmatrix}. \tag{35}$$

Eigenvalues of matrix J are

$$\lambda_2 = \begin{bmatrix} 3.0318 \\ -0.2149 \\ 0.5831 \end{bmatrix}. \tag{36}$$

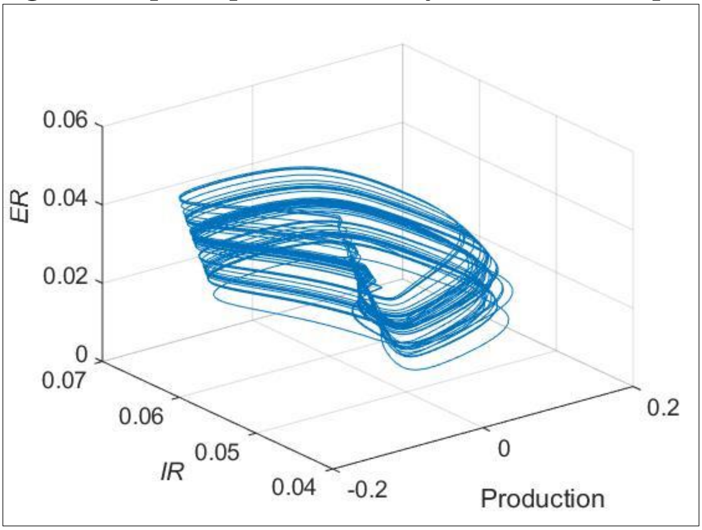
System of equations (34) thus represents a non-linear unstable system which that exhibits an interesting dynamics as we notice in Figure 3.

Fig. 2 The development of Y, IR and ER with a relatively low export multiplier



Source: Data source (authorial computation).

Fig. 3 The phase portrait of the system for a low export multiplier



Source: Data source (authorial computation).

In Figure 2 a time non-periodic or multi-periodic evolution of product, interest rate and exchange rate is displayed. In Figure 3 the attractor is shown. In both figures a more complex dynamic of economic quantities is displayed.

Now, let us assume that the export multiplier is sufficiently high, which means that a rise of exchange rate increases export and consequently export strongly raises product. The result is the decline of export-product ratio. So $x_3 < 0$.

4.2 Example 2

All parameters in this example are the same as in the first example but parameter $x_3 = -0.26$. Then the system (18) takes

$$\begin{gathered} \dot{y} = 20\left[\frac{1}{1+R}\frac{0.42}{1+e^{-10y}} - 0.26z - (0.12 + 0.8y + 1{,}6R)\right], \\ \dot{R} = 0.03 + 0.1y - 0.6R + 0.19z, \\ \dot{z} = R - 0.05. \end{gathered} \tag{37}$$

Steady state of the system is given by the following system of equations

$$\begin{gathered} 0 = \frac{1}{1+\bar{R}}\frac{0.42}{1+e^{-10\bar{y}}} - 0.26\bar{z} - (0.12 + 0.8y + 1.6\bar{R}), \\ 0 = 0.03 + 0.1\bar{y} - 0.6R + 0.19\bar{z}, \\ 0 = \bar{R} - 0.05. \end{gathered} \tag{38}$$

Steady state solution $\bar{y} = 0, \bar{R} = 0.05, \bar{z} = 0$. Is the same as in the preceding example (the examples are made with respect to simplicity and readability). The Jacobiho matrix of system (38) is

$$J = \begin{bmatrix} 4.0 & -35.8095 & -5.2 \\ 0.1 & -0.6 & 0.19 \\ 0 & 1 & 0 \end{bmatrix}. \tag{39}$$

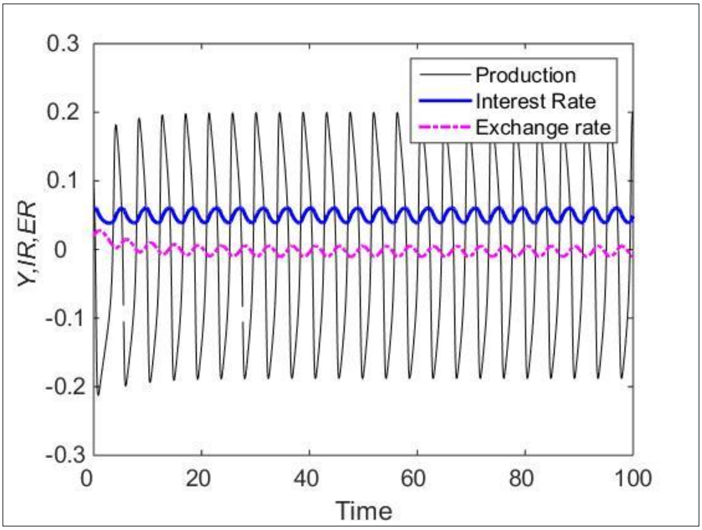
Vector of eigenvalues is:

$$\lambda_2 = \begin{bmatrix} 3.0318 \\ -0.2149 \\ 0.5831 \end{bmatrix}. \tag{40}$$

Figures 4 and 5 display the development of variables y, R, z and phase portrait of the system with relatively high export multiplier. Development of variables is periodical and the corresponding phase portrait exhibits a limit cycle.

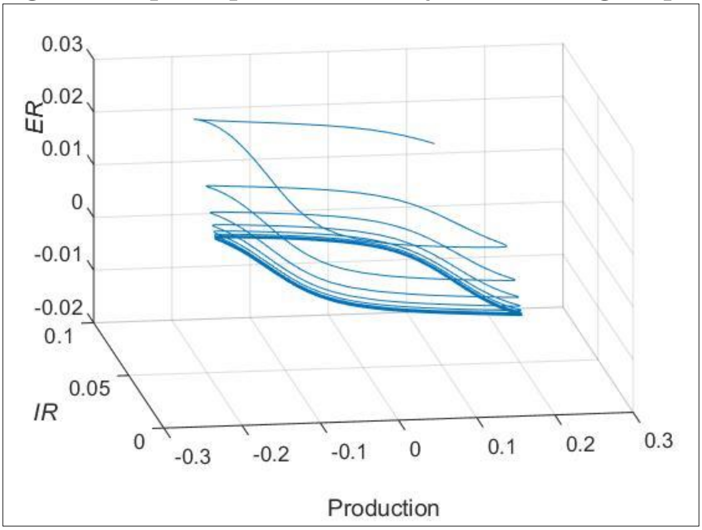
Compared to the development of variables y, R, z and phase portrait in the previous system, economic quantities in this system with high export multiplier are less turbulent. The attractor and the evolution of variables of the previous system is more complex. It can be explained by the fact that when $x_3 > 0$, it causes an increase in z. This consequently further increases $\dot{y}$ which means that the product increases more. It could be a source of turbulence. On the contrary, the case of example 2 with a higher export multiplier slows that as the production increases which reduces possibility of turbulences and hence it makes the development of the system variables more periodical.

Fig. 4 The development of Y, IR and ER with relatively a high export multiplier



Source: Data source (authorial computation).

Fig. 5 The phase portrait of the system for a high export multiplier



Source: Data source (authorial computation).

5 Conclusion

In the paper we have derived a continuous time dynamic model based on the Mundell Fleming model for open economy. We have also investigated its stability conditions by using Hurwitz stability criterion and introduced some nonlinearity into the model. After that we have generated two numerical examples. The result of this simulation shows that the dynamics of this model could considerably differ with respect to the dependence of production on net export. As we also assume that the export positively depends on exchange rate, under these assumptions we have shown on numerical examples that higher sensitivity of product on exchange rate (relatively high export multiplier) leads to less complex dynamics of system. On the other hand, relatively low export multiplier leads to more complex behaviour of the system. These results indicate that if an economy has a similar set of parameters, one can expect such dynamics from its development.

References

Dornbusch, R., 1980. Open Economy Macro-Economics. Basic Books, New York.

Flaschel, P., Franke R., Demmler,W., 1997. Dynamic Macroeconomics. MIT Press, Cambridge.

Fleming, J. M., 1962. Domestic Financial Policies under Fixed and Floating Exchange Rates. IMF Staff Papers 3, 369-379. DOI: 10.2307/3866091.

Guckenheimer, J., Holmes, P., 1986. Non–Linear Oscillation, Dynamical Systems and Bifurcations of Vector Fields. Springer Verlag, New York.

Hahn, F., 1982. Stability. Handbook of Mathematical Economics 2, 745-793. DOI: 10.1016/s1573-4382(82)02011-6.

Kodera, J., Sladký, K., Vošvrda, M., 2007. Neo-Keynesian and Neo-classical Macroeconomic Models: Stability and Lyapunov Exponents. Czech Economic Review 3, 301-311.

Mundell, R. A., 1962. The Appropriate Use of Monetary and Fiscal Policy for Internal and External Balance. IMF Staff Papers 1, 70-79. DOI: 10.2307/3866082.

Mundell, R. A., 1963. Capital Mobility and Stabilisation Policy under Fixed and Flexible Exchange Rates. Canadian Journal of Economics and Political Sciences 4, 475-485. DOI: 10.2307/139336.

Mussa, M., 1986. Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications. Carnegie-Rochester Series on Public Policy 3, 117-214. DOI: 10.1016/0167-2231(86)90039-4.

Obstfelf, M., Rogov, K., 1996. Foundations of International Macroeconomics. MIT Press**,** Massachusetts.

Sladký, K., Kodera, J., Vosvrda, M., 1999. Macroeconomic Dynamical Systems: Analytical Treatment and Computer Modelling. In: Proc. 17th Conference Mathematical Methods in Economics 1999 (J. Plešingr, ed), University of Economics, Jindrichuv Hradec, 245-252.

Takayama, A., 1994. Analytical Methods in Economics. Harvester Wheatsheaf, Hertfordshire. DOI: 10.3998/mpub.11781.

Turnovsky, S., 2000. Methods of Macroeconomic Dynamics. MIT Press, Massachusetts.

Appendix 1: Matlab script for numerical examples

```
% Matlab script for numerical examples
clear
close all
clc
ops = odeset('RelTol',1e-4,'AbsTol',[1e-4 1e-4 1e-5]);
[T,Y] = ode45(@fexample4b,[0 400],[0.1 0.06 0.02],ops);
plot(T,Y(:,1),'k-')
hold on
plot(T,Y(:,2),'b-',T,Y(:,3),'m-.','LineWidth',2)
hold off
xlabel('Time')
ylabel('\it Y,IR,ER')
legend('Production','Interest Rate','Exchange rate','Location',...
    'northeast')
axis([0 400 -0.2 0.2])
h = gca;
h.XTick = 0:100:400;
h.YTick = -0.2:0.1:0.2;
h.XTickLabel = 0:100:400;
h.FontSize = 14;
h.YTickLabel = -0.2:0.1:0.2;
figure
plot3(Y(:,1),Y(:,2),Y(:,3),'-')
xlabel('Production')
ylabel('\it IR')
zlabel('\it ER')
grid on
h1 = gca;
h1.FontSize = 14;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
function dy = fexample1b(t,y)
j1 = 0.1; j2 =0.1905;
s0 = 0.22; s1 = 0.8; s2 = 1.6;
x0 = 0.1; x3 = 0.19; %x3=0.16
l0 = 0.03; l1 = 0.1; l2 = 0.6; l3 = 0.19;
Rstar = 0.05; alpha = 20; beta = 1; m = 0.;
dy = zeros(3,1);
dy(1) = alpha*(j1*y(1) - j2*y(2) + x0 + x3*y(3) - (s0 + s1*y(1) +
s2*y(2)));
dy(2) = beta*(l0 + l1*y(1) - l2*y(2) + l3*y(3) - m);
dy(3) = y(2) - Rstar;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
function dy = fexample2b(t,y)
i0 = 0.42; i1 = 10;
s0 = 0.22; s1 = 0.8; s2 = 1.6;
x0 = 0.1; x3= -0.26; %x3=0.19
l0 = 0.03; l1 = 0.1; l2 = 0.6; l3 = 0.19;
Rstar = 0.05; alpha = 20; beta = 1; m = 0.;
dy = zeros(3,1);
dy(1) = alpha*(i0/((1 + y(2))*(1 + exp(-i1 * y(1)))) + x0 + x3*y(3) -
...
    (s0 + s1*y(1) + s2*y(2)));
dy(2) = beta*(l0 + l1*y(1) - l2*y(2)+l3*y(3) - m);
dy(3) = y(2) - Rstar;
```

```
% Matlab skript for the linear systém matrix
clear
close all
clc
i0 = 0.42; i1 = 10;
s0 = 0.12; s1 = 0.8; s2 = 1.6;
x3 = 0.19; %x3=0.19;x3=0.18; x3=0.16; x3=0.08;x3=0.03; x3=-0.1;x3=-
0.2;...
% x3 = -0.26
l0 = 0.03; l1 = 0.1; l2 = 0.6; l3=0.19;
Rstar = 0.05; alpha = 20; beta = 1; m = 0;
Rbar = 0.05; ybar = 0;
j1 = i0*i1/((exp(i1*ybar)+2+exp(-i1*ybar))*(1+Rbar));
j2 = i0/((1+exp(-i1*ybar))*(1+Rbar)^2);
```

Example 1

```
A1 = [alpha*(j1-s1),-alpha*(j2+s2),alpha*x3; beta*l1,-
beta*l2,beta*l3;0,...
1, 0];
d1 = det(A1);
lambda1 = eig(A1);
```

Example 2

```
 A2 = [alpha*((i0*i1)/((exp(i1*ybar) + 2 + exp(-i1*ybar))*(1 + Rbar)) -
s1),...
     -alpha*i0/((1 + exp(-i1*ybar))*(1 + Rbar)^2) - alpha*s2,alpha*x3;
...
     beta*l1,-beta*l2,beta*l3; 0, 1, 0];
 d2 = det(A2);
 lambda2 = eig(A2);
```