

Alternative Views on the Link between Risk Aversion and Diminishing Marginal Utility of Wealth

*Vojtěch Menzl**

Abstract:

Although the link between risk aversion and diminishing marginal utility of wealth is academically well established, theoretical discussions concerning its empirical validity remain. The presented, review-type paper aims to briefly examine theoretical roots responsible for the different views on this association in order to provide a broader perspective to alternative explanations. This latter task is assisted by comparative analysis of two recent pieces of research by Rick Falkenstein and Matthew Rabin; a duo of papers, handpicked at the author's discretion to demonstrate the convergence of alternative ideas from different authors (and backgrounds). In support of its argumentation, the paper also presents a critical overview of the equity premium puzzle as seen through the prism of behavioural finance. The main contributions of the paper include evidence-based support for the concept of relative utility and reconfirmation of the meaningful role of behavioural finance in economics and finance.

Key words: Risk Aversion; Marginal Utility; Expected Utility; Behavioural Finance; Equity Premium Puzzle.

JEL classification: D81; G11; G12.

1 Introduction

The mechanism of a trade-off between utility and risk has been pondered, discussed and studied for centuries. Over time, various concepts have been proposed to explain this link. Depending on their success in capturing reality, these theories may be ranked from simple, prescriptively nice (and removed from reality) to complex, descriptively complicated (and closer to reality). What follows is an insight into this story as seen from the perspective of two papers, chosen

* Vojtěch Menzl, Prague University of Economics and Business, Faculty of Finance and Accounting, Department of Corporate Finance and Business Valuation, W. Churchill Sq. 4, 130 67 Prague 3, Czech Republic, <vojtech.menzl@vse.cz>.

The article is processed as one of the outputs of the research project of the Faculty of Finance and Accounting at Prague University of Economics and Business *New Challenges of the Corporate Finance in the Czech Republic 2.0* registered by the Internal Grant Agency of Prague University of Economics and Business under the registration number F1/58/2021.

at the author's discretion to demonstrate that benchmarking of relative changes in value and utility apply similarly also to the perception of risk. That risk and its size gets also subordinated to some external, relative benchmark, be it a common portfolio, social status, or current wealth.

The presented, review-type paper seeks as its objective to provide critical and referenced insight into the existing discussion on the link between risk aversion and the (diminishing) marginal utility of wealth. To this end, it analytically covers two recent, topical pieces of research by Eric Falkenstein and Matthew Rabin. This particular choice of the reviewed papers was made at the sole discretion of the author on the grounds of their deemed informative value and complementary match, promising to facilitate demonstration of the remarkable difference between the *classical utility theory*, its *marginal version*, and the *relative* interpretations of both. Pointing at general validity and evidence-substantiated backing of the concept of *relative utility* is also the main contribution of the paper.

As its secondary goal, the paper aims to contribute to the discussion on the equity premium puzzle from the perspective of behavioural finance. Therefore, attention is also directed towards the evidence-based support which the selected duo of papers under review provides to the concept of relative utility specifically and to the meaningful role of behavioural finance and economics in general.

The remainder of the paper is structured as follows. The next section presents a general literature review and genesis of the development of utility and risk concepts in how their understanding has changed over time. The following two chapters devote in respective turns to the review of the aforementioned selected duo of papers. The subsequent section attempts to put the previously identified review findings into a broader perspective within the domain of behavioural finance. To this end, the paper extends its focus by a related discussion on the ways of how to deal with and explain the issue of the equity premium puzzle. The final part of the paper concludes and presents a brief authorial takeaway summary.

2 Literature Review

Since acknowledged by Bernoulli (1738) within the St. Petersburg paradox (Bernoulli apud Aase, 1998) and formulated by Gossen (1854), the concept of marginal utility (Gossen's First Law; Hagendorf, 2010, p. 5), linking income increases with ever-decreasing individual gains of personal satisfaction and happiness, sits firmly rooted in classic economic theory (e.g. Marshall, 1890). The sole increase in the expected tangible value (volume) of goods is no longer seen as a representative of the main thing, i.e. happiness (utility). In the broader perspective of theoretical advances in economic decision-making under risk

and uncertainty, this shift in attention from the expected value towards the (subjective) expected utility and its maximisation represents the second of three main development stages (see Table 1), succeeding the original phase of *expected value* maximisation (e.g. Blaise Pascal (1670) and his “wager”; Pascal apud Lengwiler, 2008)¹ and preceding the present-day phase of *generalised expected utility* maximisation (e.g. Maurice Allais, 1952 or Daniel Kahneman, 1979, apud Hey, 1997, p. ix, and others).

Tab. 1 Theoretical advances in decision-making under risk and uncertainty

Stage	Formula
I. Expected value	$E(X) = \sum_{i=1}^n x_i \cdot p_i$
II. Expected utility	$U(X) = \sum_{i=1}^n u(x_i) \cdot p_i$
III. Generalised expected utility	$U(X) = \sum_{i=1}^n v(x_i) \cdot \pi(p_i)$

Source: own compilation.

Note: x_i and p_i represent individual (monetary) outcomes and their corresponding probabilities, $u(x_i)$ are utilities² of these individual outcomes, $v(x_i)$ represent the value function v -transformed individual (monetary) outcomes and $\pi(p_i)$ stand for the weighting function π -transformed decision weights (weighted original individual probabilities for which, due to the effect of subcertainty, $\pi(p) + \pi(1-p) < 1$ (Kahneman, 1979, p. 281)).

Marginalism, an integral part of the mainstream economic theory that builds on the Marshall-formalised idea that “*the additional benefit which a person derives from a given increase of his stock of a thing diminishes with every increase in the stock that he already has*” (Marshall, 1890, p. 61), has proven a significant driving force of economic development. This progress has subsequently cleared the path to further refinements, such as the concept of marginal rates of substitution and the law of marginal substitution (Hicks in Kauder, 2016), shown in (1):

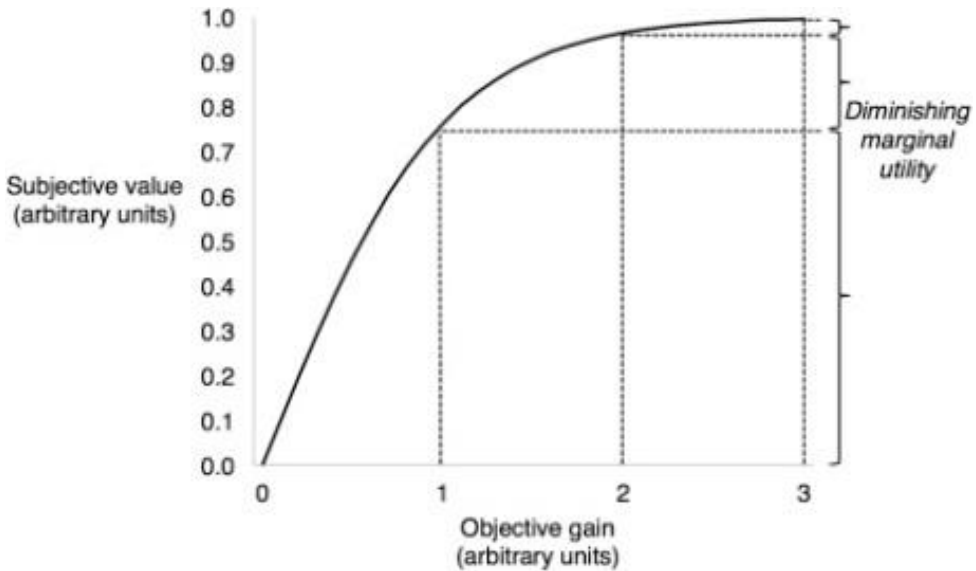
$$\frac{\text{amount of } Y \text{ gained}}{\text{amount of } X \text{ lost}} = \frac{\text{marginal utility of } X}{\text{marginal utility of } Y} \quad (1)$$

Source: Kauder (2016), p. 148.

¹ In fact it both succeeds and includes the stage one, since the expected utility theory consist of the expected value of utility (i.e. the stage one) and the assumed decreasing marginal utility; both these components in turn combine into the concept of risk aversion (Lengwiler, 2008, pp. 1–2) as foreseen already in 1738 by Daniel Bernoulli (ibid., pp. 5–6).

² The term ‘*utility*’ itself (its meaning) has gradually undergone (and still undergoes) a substantial shift from its original sense of a property which produces benefit, pleasure, good or happiness (Bentham in Encyclopedia.com, 2021) to a much broader meaning of value or preference (Moscati, 2019).

Fig. 1 Diminishing marginal utility



Source: Hirtl et al. (2016).

Equally well-established is also the link between risk aversion and diminishing marginal utility (graphically illustrated in Figure 1), which forms the basis of the general concept of risk-return theory. In this respect, the concept of utility gets extended into the expected utility theorem (e.g. von Neumann–Morgenstern’s utility theorem, von Morgenstern, 1953, respectively, Morgenstern apud Savage, 1954).

Within **expected utility theory**, the diminishing marginal utility of wealth and the concept of risk aversion get interlinked, since the latter is defined *in the context of that theory* (Yaari³, 1987). The theory of expected utility accounts for the fact that risk-averse individuals may refuse fair prospects with zero expected values. In turn, **risk aversion** implies a **concave shape** of their utility functions and therefore the diminishing marginal wealth utility. Viewed this way, risk attitude is directly related to the curvature of the utility function: while linear utility functions correspond with risk-neutral individuals, risk-seeking individuals

³ In fact, Yaari under the term “expected utility theory” refers to the diminishing marginal utility of wealth and to the risk aversion as *synonyms*, although the former reflects attitude to wealth (loss is more painful to the poor) while the latter relates to the attitude to risk (increase in uncertainty hurts). Yaari subsequently questions whether both aspects should be kept within the same theory and proposes his own theoretical approach, the “*dual theory of choice under risk*” (Yaari, 1987, p. 95).

assume convex utility functions, in further contrast to risk-averse individuals, who have concave utility functions. Therefore, the degree of risk aversion can be measured by the curvature of the utility function.

Savage (1954) has an important note on the concavity of utility functions within the understanding of the von Neumann-Morgenstern concept (apart from his say on the topic of the subjective expected utility model). Specifically, he notes that *“the law of diminishing marginal utility plays no fundamental role in the von Neumann-Morgenstern theory of utility, viewed either empirically or normatively. Therefore the possibility is left open that utility as a function of wealth may not be concave, at least in some intervals of wealth.”* (Savage, 1954, pp. 103–104).

On the other hand, the **theory of non-expected utility** within the generalised expected utility maximisation approach, of which the largest branch represents the concept of **rank-dependent** utility (Wakker, 1994, Chateauneuf, 1994), assumes that risk aversion rather separates into two distinct components: the ‘*probabilistic*’ risk aversion (convexity of the cumulative probability transformation), and the aforementioned diminishing marginal utility. This understanding of the risk-return relationship is further developed within the **behavioural approach** to economics and finance, specifically in the Prospect theory (Kahneman and Tversky, 1979) and the Cumulative prospect theory (Tversky and Kahneman, 1992), respectively, while acknowledging (and quantifying⁴) e.g. the phenomenon of **loss aversion**. These theories were in turn modified yet further to characterise utility differences independent of probabilistic risk attitudes (Wakker, 1994, p. 1).

For completeness, at least briefly should also get noted the second alternative response to the expected utility; besides non-expected utility theories, there were proposed and elaborated also **non-deterministic** approaches to choice under risk and uncertainty, of which perhaps the best known is the **random utility model** (Palma, 2008). This framework works with the additional structure of error terms and may be applied to both the EU and non-EU models (Palma, 2008, p. 270).

Nevertheless, risk aversion specified and synonymised by a strictly concave utility function (and thus a function with decreasing first derivative) still firmly remains at the core of classic economic theory.

Disagreement between the textbook theory and empirical evidence, represented e.g. by the so-called equity premium puzzle (Mehra and Prescott, 1985 and 2003)

⁴ Assuming that x_1, \dots, x_k represent gains (≥ 0) and x_{k+1}, \dots, x_n stand for losses (< 0) and using the rank-dependent model notation (including the renumbering and ranking outcomes so that $x_1 \geq \dots \geq x_n$), subsequent evaluation of utility takes the form of $\sum_{j=1}^k \pi_j U(x_j) + \lambda \sum_{j=k+1}^n \pi_j U(x_j)$, where $\lambda > 1$ is referred to as **loss aversion** (empirically suggested at ≈ 2) (Palma, 2008, p. 274).

and implausibly high levels of investor risk aversion⁵, remains a topical issue and mystery to this very day.

In 2003, the original authors of the ‘*puzzle concept*’ revisited their 1985 research conclusions in that [for the class of economies considered] *the differential between the average real annual yield of 7 per cent on the S&P 500 Index over the 90 years of 1889–1978 and the average yield on short-term debt of less than 1 per cent cannot be accounted for by models that abstract from transactions costs, liquidity constraints, and other frictions absent in the Arrow–Debreu set-up* (Mehra and Prescott, 1985, p. 145); “*the equity premium puzzle may not be why was the average equity return so high but rather why was the average risk-free rate so low*” (ibid., p. 158). Even 18 years since these original paper’s conclusions, (Mehra and Prescott, 2003) confirm their de facto status quo while commenting: “*over the long horizon the equity premium is likely to be similar to what it has been in the past and the returns to investment in equity will continue to substantially dominate that in T- bills for investors with a long planning horizon*” (ibid, p. 34).

Finally, as part of his 2007 comprehensive review of the topic, Mehra among the concluding comments to the paper once again repeats that no single explanation has succeeded in fully resolving the anomaly, although “*considerable progress has been made and the equity premium is a lesser puzzle today than it was 25 years ago*” (Mehra, 2007, p. 69).

The puzzle gets even more ‘*puzzling*’ when the U.S. annual real growth rate of per capita consumption, originally (Mehra and Prescott, 1985, p. 154) assumed to be first-order negatively serial correlated (-0.14), gets corrected in the light of findings by (Azeredo, 2007). While proposing an alternative measure to capture the extent of the serial correlation of consumption growth for the period 1899–1928, the author arrives at a positive (+0.32) correlation figure. This finding further exacerbates the original puzzle (Azeredo, 2007, p. 9).

3 Marginal Utility of Wealth, Risk and Return

The first of the two discussed papers, arbitrarily chosen on the grounds of their unorthodox approach to the concept of expected utility and risk aversion, is a rather substantial, 150-page piece of work by Eric Falkenstein (2010). Starting unaccustomedly from research conclusions, the paper (fairly unconventionally) argues that **there is no empirical risk-reward relation** and the seeming examples are sole exceptions to the general rule, explainable by liquidity premiums and

⁵ Past equity premium (i.e. return earned by a risky security in excess of that earned by a risk-free U.S. T-bill) is an *order of magnitude* greater than possibly rationalised in the context of neoclassical financial economics (Mehra, 2007, p. 2), that is, as a premium for bearing non-diversifiable risk (ibid.).

errors in measurement (Falkenstein, 2010, p. 2). This stark conclusion is nevertheless supported by 100+ pages of carefully assembled and worked out arguments. Some of the more substantial or interesting points include:

- a. Risky decisions are internalised; the null risk-return is equilibrium when people internalise risky decisions by comparing themselves to others (the proverbial '*keeping up with the Joneses*') in contrast to the standard approach to equate risk with absolute volatility of (one's own) wealth.
- b. This notion supports the so-called *Easterlin Paradox* (Easterlin, 1974, apud Falkenstein, 2010): within the given country, richer people report higher subjective well-being than poorer ones, whereas spatial (among countries) and time (within a country) comparisons reveal only minor differences in subjective well-being. The effect points towards relative vs. absolute importance of wealth for subjective happiness (well-being)⁶. Positive effects of extra (absolute) income on the quality of life are comparatively small while relative income effects are significant; within the quality of life, advanced economies should therefore focus more on inequality than on absolute wealth. The relative utility framework is supportive of this notion, while traditional, value-based utility functions, are in disagreement. Puzzling *home bias* in portfolio holdings may quite easily explain the desire to stay with one's peer group than the world's population⁷. Economically irrational preference of anti-competitive policies may also be rationally explained by valuing personal status more than the less important absolute position.
- c. Until ca. 1750, the per capita economic growth was essentially zero, but within this zero-sum world, human beings as 'social animals' needed to communicate their social status, necessary for reproduction and survival. As a consequence of evolutionary selection, this status orientation has got 'hard-wired' into our brains and the concern for status remains with us to this very day: relative wealth is more important (and valued) than its absolute size. However, formulations of relative wealth are plentiful, ratios or differences, relative to the median or the arithmetic mean, etc.
- d. The relative risk model dominates the absolute risk model.

⁶ Falkenstein uses an anecdotal definition of 'being wealthy' in reference to a man who earns USD 100 more than his wife's sister's husband; reinforcing again the relative nature of well-being (financial or other).

⁷ Formally, explanations of *home bias* include more factors, such as ambiguity attitudes (Palma, 2008, p. 275), familiarity, foreign currency risk, political risk, tax purposes (mainly double taxation), and possibly more.

- e. Within the conventional risk-return model, the gradual flattening of the utility curve with the rise in wealth implies that people are almost indifferent both to wealth and risk.
- f. It seems implausible that risk preferences have declined over the past centuries; the risk-free rate would have adjusted accordingly – and this has not been the case. Instead, there is a case for ‘*constant relative risk aversion*’ (p. 74), i.e. ‘risk’ is relatively the same, regardless of how much wealth one owns.
- g. The modern notion of risk is associated with the function of $U(c) = \frac{c^{1-a}}{1-a}$, where c represents consumption and a denotes the risk aversion constant (usually between 3 and 10 for the function to imply consistent interest rates as seen over the past millennium).
- h. Despite being intuitively more subjective than the concept of risk, the notion of e.g. beauty still holds certain subjective preferences which may be mathematised and ranked. While arguably less subjective than the notion of beauty, the concept of risk, nevertheless, remains infinitely more difficult to identify.
- i. Although risk as a practical matter is insanely subtle, it allows being benchmarked (e.g. the Sharpe ratio, Sharpe, 1994). In turn, the risk is relative to a benchmark.
- j. Provided all investors act ‘*as if*’ they benchmark to aggregate indices, risk will not be in equilibrium.
- k. Utility is understood as a *status* function, value of one’s wealth relative to that of his or her peers, where only deviations from the agreed consensus are ‘*risky*’.
- l. Such a ‘risk’ may be avoided when everyone holds an identical market portfolio (similarly to the diversifiable risk within Markowitz’s Modern Portfolio Theory (Markowitz, 1952a) and the world of absolute returns, respectively) possible to be priced with no further justification.
- m. This irrelevance of risk to return is implied by the status-conscious investor’s benchmarking against others; risk is “*simply an allocation of an ‘unusual’ amount of wealth to any asset that would generate a significant deviation from the market portfolio*” (p. 137).
- n. Too little exposure to certain asset classes is viewed the same as too much of it.
- o. The result is an empirical implication: all assets have the same expected return (rather than variable risk premia); the highest volatility assets, however, may

experience a rational decline in their expected returns when these become subjects to overconfidence.

- p. Empirical failure of [CAPM's] 'beta' is a well-known fact (the in- paper presented model explains this failure, similarly also e.g. Sinha, 1994).
- q. The relative-status oriented utility function corresponds to CAPM- and APT-consistent factor model, except that the risk premia are zero. 'Beta' is still descriptive of the relative volatility, generating normative predictions for volatility minimisation.
- r. There is no robust cross-sectional return to any 'beta' and no upward sloping security market line; the portfolio optimisation algorithm for an investor with typical preferences is thus 'trivial': allocate assets to standard categories of the conventional wisdom.

Saving a more detailed discussion for later, let us directly move to the second chosen paper.

4 Diminishing Marginal Utility of Wealth and Risk Aversion

The topic of the second selected paper, researched by Matthew Rabin (2000a) and subsequently referred to as the *calibration theorem* (Rabin, 2000b, Bombardini, 2010), has been subjectively chosen for its fit with the preceding contribution.

Moreover (and outside authorial intentions when deciding on the most suitable review candidates), its author gets also cited by Falkenstein (2010): "*Mathew Rabin won a MacAurthur Foundation genius grant ... for showing (Rabin, 2000a) that one can apply the fact that concave functions to show that if one has the above utility, and one chooses to turn down a 50/50 bet to lose \$10 or gain \$11, one would not accept a bet to lose \$100 and win an infinite amount of money. This absurdity highlights a profound problem with our fundamental conception of utility.*" (Falkenstein, 2010, pp. 72–73).

Yet another, admittedly *ex-post* reason for picking up this specific paper by Rabin, is the rather limited coverage of this topic within the *Scopus* citation database; my initial search query has returned only 24 source-hits, from which the covered Rabin (2000a) appears the most suitable paper for the intended comparison (followed by the previously mentioned Wakker (1994) and Chateauneuf (1994), who, however, concentrate rather specifically on the rank-dependent utility domain).

The paper by Rabin relates directly to Falkenstein (2010) in that **risk aversion is pervasively explained by** the assumption that people as economic subjects have generally **diminishing marginal utility of wealth**. Although explaining probably much of our aversion towards large-scale financial risks that threaten our lifetime

wealth, Rabin (2000a) considers it utterly implausible that the sufficient *and* necessary concavity of the utility-of-wealth function (and thus the diminishing marginal utility of wealth) should be the *sole* explanatory factor to risk aversion within the expected-utility framework. In his opinion (substantiated further in his text), “*any utility-of-wealth function that does not predict absurdly severe risk aversion over very large stakes predicts negligible risk aversion over modest stakes*” (Rabin, 2000a, p. 1).

While Arrow (1971) claims that expected-utility maximising subjects always want to take a sufficiently small stake in any positive-expected value bet (“*a risk-averter takes no part of an unfavourable or barely fair gamble; on the other hand he always takes some part of a favourable game; intuitively, risk can be reduced to arbitrarily small proportions by making the amount purchased of the risky asset small, while the expected profit per unit investment is positive and constant. ... for small amounts of risk, the utility function is approximately linear, and risk aversion disappear*” (Arrow, 1971, p. 100)), according to Rabin (2000a,b) this approximate risk neutrality also extends to quite sizable and economically important stakes.

In reformulation, adopting expected utility theory for the purposes related to explaining substantial risk aversion often results in misleading conclusions, since “*the theory actually predicts virtual risk neutrality*” (ibid., p. 1); cf. “*there is no empirical risk-reward relation*” (Falkenstein, 2010).

In his award-winning contribution, Rabin goes on to demonstrate – for any concave utility function within the expected utility framework – that even small risk aversion over modest bets implies an absurd degree of risk aversion over large stakes. Furthermore, there are no parametric assumptions attached to such a utility function except for its increasing and concave shape. “*If an expected-utility maximizer always turns down modest-stakes Gamble X, she will always turn down large-stakes Gamble Y*” (Rabin, 2000a).

This arguably leads to absurd conclusions: provided the given subject turns down a 50/50 gamble of winning USD 110 vs. losing USD 100, the very same subject will be expected to turn down a 50/50 bet of losing USD 1,000 and winning *any* (unlimited) sum of money! Turning down a 50/50 loss of USD 1,000 and gain of USD 1,050 bets implies avoiding 50/50 prospects of losing USD 20,000 and gaining *any* sum. These are just several examples of many which Rabin summarises and tabularizes in his paper. The culprit is the assumed rate by which the value of money diminishes; when iterated towards unlimited gains, these are assumed to have a zero subjective value.

Specifically, rejecting a 50/50 bet to lose USD 10 or win USD 11 implies the non-strict inequality (2), where $U(\cdot)$ stands for the respective utility:

$$U(W+11) - U(W) \leq U(W) - U(W-10) \quad (2)$$

Therefore, each dollar between W and $W+11$ is on average valued at most $10/11$ as much as the average dollar value in the range W and $W-10$.

Assuming an increase in wealth to $W+21$ and preserving the original aversion to lose USD 10 in exchange to win USD 11, the subject values $(W+21)+11=W+32$ in dollars by at most $10/11$ as the dollar value of $(W+21)-11=W+11$. This means that $W+32$ dollars are valued at most $10/11 \times 10/11 \sim 5/6$ as much as $W-10$ dollars. By the same logic, the $W+880$ th dollar is valued by most at $1/2,000$ as much as the $W-10$ th dollar. Iteration this way results in absurd conclusions on the rejection or acceptance of bets associated with varying payoffs. Respectively, it points at the fact that “*aversion to modest-stakes risk has nothing to do with the diminishing marginal utility of wealth*” (Rabin, 2000a).

While expected utility proves useful in explaining large-stake insurance, it is by no means a good tool to explain risk attitudes towards modest stakes. Results of laboratory experiments are mostly irrelevant for empirical, real-world applications (Rabin, 2000a, p. 5)

Moreover, the expected-utility theory implies that if a person turns down a particular gamble, he or she should also turn down an offer to play ($n>1$) of these games (Samuelson, 1963, apud Rabin, 2000a, p. 6, apud Thaler et al., 1997, p. 649). This conclusion is highly counter-intuitive: aggregating 10,000 individual 50/50 bets to lose USD 100 or win USD 200 leads to a highly *rational acceptance* of such a lottery based on the mean value of the prospect.

Concerning this paradox, Rabin and Thaler (2001) likens us, users of established though seemingly imperfect theories, to the buyers who have purchased a ‘*dead parrot*’ (in reference to the *Monty Python’s* sketch); for while trying to return a faulty purchase, we are only to face a long list of arguments why our claims are without merit (the clearly dead parrot/theory is “*merely resting, being shagged out after a long squawk, prefers resting on its back, and that it is pining for the fjords of Norway*” ... “*This is an ex-parrot!*” (Rabin and Thaler, 2001, p. 230)). In this context, as the authors put it, “*it is time for economists to recognize that expected utility is an ex-hypothesis*” (ibid).

5 Relative Utility and Behavioural Finance

Since both presented papers touch upon the concept of behavioural economics and finance, perhaps fitting is to present also few comments in this regard.

Rabin (2000a) makes the explicit case of behavioural finance in that *loss aversion* (Kahneman and Tversky, 1979) may stand responsible for the generally more pronounced aversion to losses relative to the status quo when compared to the possibility of winning. More generally, personal utility is determined by the changes in wealth (i.e. relatively) more than by the values of absolute wealth itself⁸ (cf. Falkenstein, 2010, *Easterlin Paradox*, and the bullet point ‘b’ above, respectively). Though formally related to the equity premium puzzle and discussed later, Constantinides (1990) on a similar note assumes within his model of *habitat persistence* that utility is defined over the difference between the current consumption and lagged past consumption (Constantinides in Mehra, 2007, p. 27).

Moreover, the concept of behavioural finance also seeks answers to the aforementioned “*equity premium puzzle*” in that such a risk aversion can be explained using the loss-averse preferences in assessing gains and losses over a shorter rather than longer time horizon (for which individuals effectively invest). Behaviourally realistic alternatives are thus the likely candidates to be considered in permitting further improvements in the economic analysis. Some of these candidates will be presented or listed further in the text.

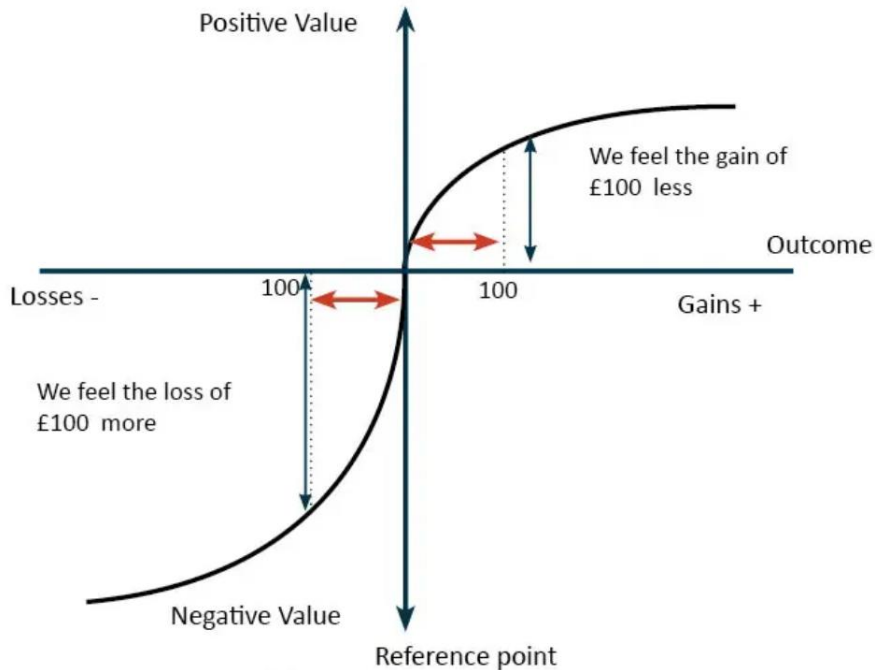
Within the Prospect and Cumulative prospect theory, respectively, the domain of utility gets represented by the *value function* component⁹, depicted in Figure 2¹⁰.

⁸ Watch MIT LFE (2017) for Harry Markowitz recalling the origins of the Prospect Theory and how struggling Tversky found an explanation to his and Kahneman’s empirical findings in the then 25 years old paper by Markowitz (1952b): “*If you want to explain actual behaviour, do not attach utility to wealth, attach it to change in wealth*”; the interview also touches upon the origins of the convex-concave shape of the value function curve and the reference point of inflection, called “current wealth” by Tversky and “customary wealth” by Markowitz (see later) (MIT LFE, 2017, time 13:47–14:59).

⁹ The concept of the Prospect theory works with two main components: the value function, concave for gains and convex for losses (with a steeper slope for losses, as shown in Fig. 3), and the inverse S-shape weighting function (Kahneman and Tversky, 1979).

¹⁰ It may be fitting to add that similarly shaped utility-value curve has been proposed by Harry Markowitz already in 1952 (Markowitz, 1952b, p. 154).

Fig. 2 Value function

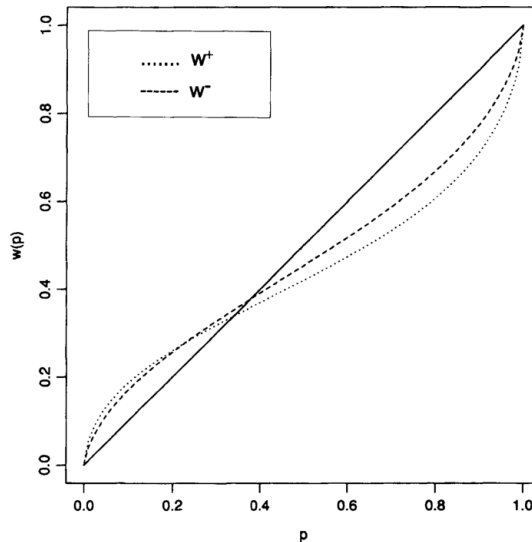


Source: Economicshelp.org (2021).

As Figure 2 shows, the '*pain*' associated with the given loss is felt more acutely than any '*happiness*' related to gaining exactly the same monetary amount (outcome) relative to the reference (point of inflection). Worth noting is the similarity between Figure 1 and Figure 2 concerning the shape of the respective curve in quadrant I (the utility function- and value function-transformed monetary outcomes, respectively). The diminishing nature of marginal values gets clearly shared between the expected and generalised expected utility. Where these theories differ, however, is in the department of the corresponding probabilities: while the expected theory assumes the linear contribution of individual probabilities to the product of $u(x_i)$ and $p(x_i)$ (for an explanation of the notation, refer to the footnote of Figure 1), the generalised expected theory (represented in our case by the Cumulative prospect theory) introduces one addition refinement in the form of the probability weighting function (Tversky, 1992) which deviates from the linear representation of probabilities under the concept of expected utility theory (in overweighting

small probabilities while underweighting large chances). Examples of the possible shapes assumed by the weighting function¹¹ are shown in Figure 3.

Fig. 3 Weighting function



Source: Tversky (1992).

In fact, since 1985 when the original topic of the *puzzle* was noted, numerous authors (see e.g. Mehra, 2007 for an excellent literature review, resp. for authors who take the findings in Mehra and Prescott (1985) as given), of which several are high-ranking proponents of the behavioural approach to finance and economy, have put forward theories that aim to address this issue. In 1995, Shlomo Benartzi and Richard Thaler introduce the term “*myopic loss aversion*” in reference to the combination of the short-term horizons and a strong distaste for losses. By adopting the behavioural concepts of *mental accounting* and *loss aversion*, the solution to the puzzle is seen in the combination of high sensitivity to losses with a prudent tendency to frequently monitor one’s wealth (Benartzi and Thaler, 1995, p. 22). Although unable to come up with direct evidence in support of such loss aversion, the given paper and its enthusiasm (“*myopic loss aversion deserves*

¹¹ For demonstrating purposes, Figure 3 depicts an updated form of the weighting function as published in (Tversky, 1992). Nevertheless, its original version in Kahneman (1979) still retains certain comparative theoretical niceties, notably allowing for **empirical discontinuity** at extreme ends due to the categorical „mismatch“ between probable and certain outcomes, the latter associated with $p=0$ and $p=1$, respectively: „The sharp drops or apparent discontinuities [of p] at the endpoints are consistent with the notion that there is a limit to how small a decision weight can be attached to an event, if it is given any weight at all. ... This quantal effect may reflect the categorical distinction between certainty and uncertainty.“ (Kahneman, 1979, p. 282).

some consideration as a possible solution to Mehra and Prescott's *fascinating puzzle*", *ibid.*, p. 23) has cleared the path to the follow-up research (Thaler et al., 1997). This time, the gap in terms of the missing experimental proof has been seemingly filled (*ibid.*, p. 648). Within the experimental setup consisted of 80 undergraduate students at UC Berkeley, tested were two myopic loss aversion-implied predictions, specifically: (a) final allocations to bonds should fall as the length of the evaluation period increases, and (b) the allocation to bonds should fall when returns are transformed to eliminate losses (*ibid.*, p. 654). The results were quite strong in that adding a constant to returns has increased the allocation to stocks by over 30 per cent (*ibid.*, p. 658) despite letting the subjects know that the high returns were in part due to the high level of intra-economy inflation. This is seen as a laboratory-induced *money illusion* and possibly also a partial explanation of one aspect of the equity premium puzzle, namely the "*risk-free rate puzzle*". In Weil (1989), the equity premium puzzle gets enhanced by another, related puzzle, when it is shown that "*the solution to the 'equity premium puzzle' documented by Mehra and Prescott (1985) cannot be found by simply separating risk aversion for intertemporal substitution*" and "*the direction of 'more realism' adds, if anything, a 'risk-free rate puzzle'*"¹², to Mehra's and Prescott's '*equity premium puzzle*' (Weil, 1989, pp. 14–15). Because risk-free rates were in real terms hardly positive, the standard model struggles to explain the reluctance of investors to take on (some) risk, respectively, their preference to save instead at very low real interest rates. The *money illusion* may present fixed-income assets as "*no loss*" investments, an attribute missing from equities (Thaler et al., 1997, p. 658). From the behavioural perspective, the situation may also be viewed through the prism of the phenomenon of *framing*, i.e. the possibility that certain disadvantages can be framed as either *cost* or *loss* (such as in the case of a purchase of insurance policy) (Kahneman, Tversky, 1984, p. 349). "*However, it has not been the case that inflation has been kind to stock market returns, at least in the short run. Clearly, this is a complex question that deserves more attention.*" (Thaler et al., 1997, p. 658).

The scope of academic explanatory approaches, which get in Mehra (2007) broadly split into the risk-based (9 representatives, *ibid.*, pp. 25–51) and non-risk based (5 representatives, *ibid.*, pp. 52–68), is far wider than our limited spaces allows to cover and includes e.g. the previously mentioned Constantinides (1990), who incorporates *habit formation* and adopts the phenomenon of *habit persistence* to show that "*equity premium puzzle is [allegedly] resolved in a rational*

¹² A new puzzle, focused on the risk-free rate: "*why is it, if consumers are so averse to consumption fluctuations, that the risk-free rate is so low?*" (Weil, 1989, p. 12). In other words, since the U.S. short-term real rate averages less than 1 per cent, the observed equity premium would require an unacceptably high risk-free rate (Mehra, 2007, p. 19).

expectations model, once we relax the time separability of preferences and allow for adjacent complementarity in consumption, a property known as habit persistence” (ibid., p. 520). For all the attitudes which do not make it into our paper, at least passing reference should be given to the *Prospect theories*, specifically to the notion of ‘*narrow framing*’ which assumes that investors’ utility is defined only over the equity gains (losses), as opposed to aggregate gains (losses), relative to the total wealth (Mehra, 2007, p. 46).

Of interest is also the attractiveness of crossovers between economics and psychology (such as in behavioural economics) to other research domains involved in decision-making. In medicine (neurobiology) it is *neuroeconomics* (Glimcher and Fehr, 2014) and its focus on *ordering*, a form of transitivity requirement associated with von Neumann–Morgenstern axioms of expected utility. Ordering requires that if one chooses e.g. 100 today over 200 in one month time and 200 in one month time over 300 in two month time, that subject would also choose 100 today over 300 in two month time. This attitude corresponds with the valuation in intertemporal choice, i.e. exponential (or hyperbolic¹³) discounting of time-dependent rewards (ibid., p. 174). Time discounting, i.e. choosing between small immediate and larger delayed outcomes gets within *neuroeconomy* viewed also from the medical (and pharmacology) perspective: *impatient choice* (preference for small immediate rewards over larger delayed rewards) is studied (and driven) by way of laboratory administered dopamine (amphetamine, methylphenidate) and serotonin stimulants (ibid., pp. 263–266).

Viewed from the perspective of financial theory, discounting negative cash flows (including the pain of loss as seen in behavioural economics) may also call for a conceptually different approach when compared to their positive counterparts (Menzl, 2020).

In closing, there seems to be a valid point in that the Prospect theory and behavioural finance have their say within the adoption of non-expected utility function into the domain of real-world economy and the actual behaviour of economic agents.

6 Summary and Conclusions

The presented contribution brings attention to evidence-based conclusions as presented in the original two papers under review. These are unanimously supportive of the validity of the concept of relative utility to the point when the expected utility’s prescription of consumer risk aversion over modest to not-so-modest stakes appears *counter-intuitive*. Not so much because people would

¹³ Whether hyperbolic discounting should be classified within behavioural or neoclassical economics remains unclear as this behavioural phenomenon is currently under wide adoption by economists across the spectrum (Luoto and Carman, 2014, p. 3).

have poor intuitions about non-zero probabilities, but because “*it is crazy*” (Rabin, 2000a, p. 7). The main similarities of the three utility concepts introduced in Table 1 may be summarised as follows:

Tab. 2 Summary of the main concepts/stages within decision-making under risk and uncertainty

Stage	Ease of use/ unequivocal for decision-making	Risk aversion covered	Loss aversion and framing of gains/losses covered
Expected value	EASY / YES	NO	NO
Expected utility	FAIRLY EASY / INPUT-DEPENDENT	YES	NO
Generalised expected utility	DIFFICULT / INPUT- DEPENDENT	YES	YES

Source: own compilation.

From the summary Table 2 it becomes clear that apart from its mathematical ease (no subjectively estimated inputs are necessary) and unequivocal outcomes (welcomed in any formal decision-making tasks), the concept of expected value (EV) has little to offer in terms of subjective reality¹⁴. The concept of expected utility (EU) builds on the theory of EV and extends it with non-linear changes in the utility-transformed values (i.e. diminishing marginal utility). The generalised EU, represented by the (Cumulative) Prospect theory, advances even further and apart from the value component modifies (using the value function transformation, see Figure 2) also the decision weights (which replace the standard probabilities, as seen in Figure 3). This brings along undisputed benefits and realism for individual decision-making tasks, although the price to be paid is reduced transparency (predictability) of the decision-making processes and a necessity to subjectively estimate additional formula inputs (see Table 1).

The paper also points at the current (though in the broader perspective of academic research not as “*modern*” as sometimes implied) gradual shift from the *prescriptive*, mathematically and logically “*nice*” yet often oversimplified ways of specifying reality and how the world “*ought*” to work, towards more *descriptive* and reality-oriented models (Tversky and Kahneman, 1992, et al.). The actual input-output validity checks of any suggested models may in turn serve as the reality check of these models’ predictions.

¹⁴ Of course, EV, the “*normatively prescribed multiplication rule for integrating probability and value of each individual outcome*” (Schlottmann, 2001, p. 1), remains at the core of normative standards taught from the primary schools and understood even before it. As such, it gets deep into our intuition, e.g. “*It appears that children acquire the EV intuition in their everyday life, a concept functional by the time they start school and before formal instruction with probabilities.*” (ibid., p. 34)

The concept of utility attached to certain, absolute stock of wealth and its changes (classical and marginal versions of the utility theory, respectively) gets contrasted with the peer-group (i.e. relative) perceptions of the same.

Using the examples of *Easterlin Paradox* and social benchmarking, the paper argues (and apart from its own authorial inputs broadly builds on the respective supporting arguments of the reviewed contributions) that the relative concept of utility holds rather more explanatory power. Benchmarking of the relative changes in value and utility apply in a similar way also to the perception of risk in that its absolute size and changes are subordinated to some exogenous, relative benchmark (e.g. common portfolio).

Within the puzzles of equity and risk-free rates of return, the contribution argues that attitude to risk is also a behaviourally-related phenomenon and its study fits well within the behavioural branch of economy and finance. According to the author who has originally brought the *equity premium puzzle* to the limelight of academic attention, there is still no clear resolution to this issue, although the subject is “*a lesser puzzle today than it was 25 years ago*” (Mehra, 2007, p. 69).

Arguably, the reviewed duo of papers presents interpretations that are rather descriptive and more research on the actual (live) data is certainly due in order to draw better-supported conclusions. The presented paper does not aim this high; instead, its aim and intended contribution were to draw the parallel among selected, more recent pieces of research, to serve as a possible thought material.

Clearly, there still remains much to be uncovered within this promising (and even ‘*fascinating*’, were the adjective not been previously taken by Benartzi and Thaler, 1995) domain at the border of economy and psychology. Hopefully, even more research will follow into this domain soon.

References

- Aase, K., 1998. The St. Petersburg Paradox. Norwegian School of Economics and Business Administration. Department of Finance and Management Science.
- Allais, M., 1952. The Foundations of a Positive Theory of Choice Involving Risk and a Criticism of the Postulates and Axioms of the American School. Apud Hey, 1997.
- Arrow, K., 1971. Essays in the Theory of Risk-Bearing. Markham Publishing Company.
- Azeredo, F., 2007. The Equity Premium: A Deeper Puzzle. UC Santa Barbara Departmental Working Papers. Available from: <escholarship.org/uc/item/6ks5p6v5>. [21 November 2021].
- Benartzi, S., Thaler, R., 1995. Myopic Loss Aversion and the Equity Premium Puzzle. No 4369, NBER Working Papers from National Bureau of Economic Research, Inc.
- Bernoulli, D., 1738. Specimen theoriae novae de mensura sortis. Comm. Acad. Sci. Imp. Petropolitanae 5, pp. 175–192. Apud Aase, 1998. Apud Lengwiler, 2008.
- Bombardini, M., Trebbi, F., 2012. Risk Aversion and Expected Utility Theory: An Experiment with Large and Small Stakes. Journal of the European Economic Association, vol. 10(6), pp. 1348–1399.
- Chateauneuf, A., Cohen, M., 1994. Risk seeking with diminishing marginal utility in a non-expected utility model. J Risk Uncertainty 9, 77–91.
- Easterlin, R., 1974. Does Economic Growth Improve the Human Lot? Some Empirical Evidence. In: David, R. and Reder, R., Eds., Nations and Households in Economic Growth: Essays in Honor of Moses Abramovitz, Academic Press, New York.
- Economicshelp.org, 2021. [Fig. 2]. Available from: <economicshelp.org/blog/glossary/prospect-theory>. [21 November 2021].
- Encyclopedia.com, 2021. Utility. International Encyclopedia of the Social Sciences. Available from: <encyclopedia.com/social-sciences/applied-and-social-sciences-magazines/utility>. [21 November 2021].
- Falkenstein, E., 2009. Risk and Return in General: Theory and Evidence. Available from: <academia.edu/25058448/Risk_and_Return_in_General_Theory_and_Evidence>. [21 November 2021].
- Glimcher, P., Fehr, E. (ed.), 2014. Neuroeconomics: Decision Making and the Brain. 2nd edition. London: Elsevier Academic Press.
- Gossen, H., 1854. Entwicklung der Gesetze des menschlichen Verkehrs und der daraus fließenden Regeln für menschliches Handeln. 1st ed.

Braunschweig: Verlag von Friedrich Vieweg und Sohn. Apud Hagendorf, K., 2010. A Critique of Gossen's Fundamental Theorem of the Theory of Pleasure.

Hagendorf, K., 2010. A Critique of Gossen's Fundamental Theorem of the Theory of Pleasure. Available from: <ssrn.com/abstract=1615522>. [21 November 2021].

Hey, J. (ed.), 1997. The Economics of Uncertainty. Vol. 1, Risk. The international library of critical writings in economics; 73. Cheltenham: Edward Elgar.

Hirt, E., Clarkson, J., Jia, L., 2016. Self-Regulation and Ego Control. San Diego: Academic Press.

Kahneman, D., Tversky, A., 1979. Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, Econometric Society, vol. 47(2), pp. 263–291.

Kahneman, D., Tversky, A., 1984. Choices, Values, and Frames. *American Psychologist*, vol. 39(4), pp. 341–350.

Kahneman, D., Tversky, A., 2000. Choices, Values, and Frames. Cambridge: Cambridge University Press.

Kauder, E., 2016. A History of Marginal Utility Theory. Princeton Legacy Library, Princeton University Press.

Lengwiler, Y., 2008. The Origins of Expected Utility Theory.

Luoto, J., Carman, K., 2014. Behavioral Economics Guidelines with Applications for Health Interventions. Inter-American Development Bank, Technical Note 665.

Markowitz, H., 1952a. Portfolio Selection. *The Journal of Finance*, vol. 7(1), pp. 77–91.

Markowitz, H., 1952b. The Utility of Wealth. *Journal of Political Economy*, vol. 60(2), pp. 151–158.

Marshall, A., 1890. Principles of Economics. London: Macmillan and Co., 8th ed.

Mehra, R., 2007. The Equity Premium Puzzle: A Review. *Foundations and Trends in Finance*, vol. 2(1), pp. 1–81.

Mehra, R., Prescott, E., 1985. The Equity Premium: A Puzzle. *Journal of Monetary Economics*, vol. 15(2), pp. 145–161.

Mehra, R., Prescott, E., 2003. The Equity Premium in Retrospect. NBER Working Papers 9525, National Bureau of Economic Research, Inc. Available from: <papers.ssrn.com/sol3/papers.cfm?abstract_id=383441>. [21 November 2021].

Menzl, V., 2020. Time Value of Risk: Value Treatment of Negative Risky Cash Flows. In preparation for publishing, available from: <hippocampus.si/ISBN/978-961-293-025-7/125.pdf>. [21 November 2021].

MIT LFE, 2017. In Pursuit of the Perfect Portfolio: Harry M. Markowitz. MIT Laboratory for Financial Engineering. Available from:

<[youtube.com/watch?v=wdeoIPCftDU&ab_channel=MITLaboratoryforFinancialEngineering](https://www.youtube.com/watch?v=wdeoIPCftDU&ab_channel=MITLaboratoryforFinancialEngineering)>. [21 November 2021].

Moscatti, I., 2019. *Measuring Utility: From the Marginal Revolution to Behavioral Economics*. New York, NY: Oxford University Press.

Neumann, J., Morgenstern, O., 1953. *Theory of Games and Economic Behaviour*. Princeton: Princeton University Press.

Palma, A. et al., 2008. Risk, Uncertainty and Discrete Choice Models. *Marketing Letters* vol. 19(3–4), pp. 269–85.

Pascal, B., 1670. *Pensées*. Apud Lengwiler, Y., 2008. *The Origins of Expected Utility Theory*.

Rabin, M., 2000a. Diminishing Marginal Utility of Wealth Cannot Explain Risk Aversion. Chapter in *Choices, Values, and Frames*, Daniel Kahneman and Amos Tversky (Eds.), New York: Cambridge University Press.

Rabin, M., 2000b. Risk Aversion and Expected-Utility Theory: A Calibration Theorem. *Econometrica*, vol. 68(5), pp. 1281–1292.

Rabin, M., Thaler, R., 2001. Anomalies – Risk Aversion. *Journal of Economic Perspectives*, vol. 15(1), pp. 219–232.

Samuelson, P., 1963. Risk and Uncertainty: A Fallacy of Large Numbers. *Scientia* vol. 98, pp. 108–113.

Savage, L., 1954. *The Foundations of Statistics*. John Wiley & Sons.

Schlottmann, A., 2001. Children's probability intuitions: Understanding the expected value of complex gambles. *Child Development*, vol. 72(1), pp. 103–122.

Sharpe, W., 1994. The Sharpe Ratio. *The Journal of Portfolio Management* Fall 1994, vol. 21(1), pp. 49–58.

Sinha, T., 1994. Prospect Theory and the Risk Return Association: Another Look. *Journal of Economic Behavior & Organization*, Elsevier, vol. 24(2), pp. 225–231.

Thaler, R., Tversky, A., Kahneman, D., Schwartz, A., 1997. The Effect of Myopia and Loss Aversion on Risk Taking: An Experimental Test. *The Quarterly Journal of Economics*, vol. 112(2), pp. 647–661.

Tversky, A., Kahneman, D., 1992. Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty*, vol. 5, pp. 297–323.

Wakker, P., 1994. Separating Marginal Utility and Probabilistic Risk Aversion. *Theory and Decision*, vol. 36, pp. 1–44.

Menzl, V.: *Alternative Views on the Link between Risk Aversion and Diminishing Marginal Utility of Wealth*

Weil, P., 1989. The Equity Premium Puzzle and the Risk-Free Rate Puzzle. No 2829, NBER Working Papers from National Bureau of Economic Research, Inc.

Yaari, M., 1987. The Dual Theory of Choice under Risk. *Econometrica*, vol. 55(1), pp. 95–115.